MTH 310 HW 8 Solutions

April 2, 2016

Section 4.4, Problem 1

Find polynomials $p(x) \in \mathbb{Z}_2[x]$ and $q(x) \in \mathbb{Z}_3[x]$ that induce the zero function in their respective fields.
Answer. If $p(x) = x^2 + x, p(0) = 0$ and $p(1) = 0$ so p induces the zero function on $\mathbb{Z}_2$. Similarly if $q(x) = x^3 - x$ then in $\mathbb{Z}_3$ $q(0) = q(1) = q(2) = 0$.

Section 4.4, Problem 5

Let $\mathbb{F}$ be a field. Show $(x-1)|a_nx^n + ... + a_1x + a_0$ if and only if $a_n + ... + a_1 + a_0 = 0$.
Answer. Assume $(x-1)|a_nx^n + ... + a_1x + a_0$. Then for some polynomial $q(x) \in \mathbb{F}[x], (x-1)q(x) = a_nx^n + ... + a_1x + a_0$. Therefore $a_n + ... + a_1 + a_0 = (1-1)q(1) = 0$.

Assume $a_n + ... + a_1 + a_0 = 0$. Then if $p(x) = a_nx^n + ... + a_1x + a_0, p(1) = 0$, so $(x-1)|a_nx^n + ... + a_1x + a_0$ by The Factor Theorem.

Section 4.5, Problem 2

Show that if p is prime then $\sqrt{p} \notin \mathbb{Q}$.
Answer. By definition, $\sqrt{p}$ is the positive element $x \in \mathbb{R}$ such that $x^2 = p$. Therefore $x^2 - p = 0$. This is a polynomial with integer coefficients, so the rational root theorem applies which says that if $x \in \mathbb{Q}$, $x \in \{1, -1, p, -p\}$ since p is prime. Therefore since $(\pm p)^2 - p \neq 0 \neq (\pm 1)^2 - p$, $x \notin \mathbb{Q}$.